Algunos ejercicios que fueron preguntas en instancias de evaluación sobre el nivel con que llegan a la FING- **Corrección**

1

$$\frac{5^{\frac{5}{4}} \cdot 8^{1/2}}{4^3 \cdot \sqrt{20}^{1/2}} = \frac{5^{\frac{5}{4}} \cdot (2^3)^{1/2}}{2^6 \cdot \left[(20)^{1/2}\right]^{1/2}} = \frac{5^{\frac{5}{4}} \cdot 2^{3/2}}{2^6 \cdot (5 \cdot 4)^{1/4}} =$$

$$= \frac{5^{\frac{5}{4}} \cdot 2^{3/2}}{2^6 \cdot 5^{1/4} \cdot 2^{2/4}} = \frac{5^{\frac{5}{4}} \cdot 2^{3/2}}{2^6 \cdot 5^{1/4} \cdot 2^{1/2}} = 5^{\frac{5}{4}-\frac{1}{4}} \cdot 2^{\frac{3}{2}-6-\frac{1}{2}} =$$

$$= 5^{\frac{4}{4}} \cdot 2^{\frac{2}{2}-6} = 5 \cdot 2^{-5} = \boxed{\frac{5}{2^5}}$$ opción (b)

2

$$\begin{array}{l|l} N & 15 \\ \hline 12 & q \end{array} \qquad \begin{array}{l|l} N & 5 \\ \hline r & q' \end{array} \Rightarrow N = 5q' + r$$

$$N = 15 \cdot q + 12 = 5 \cdot 3q + 10 + 2 = 5\underbrace{(3q+2)}_{q'} + \underbrace{2}_{r}$$

$r = 2 \Rightarrow$ opción (a)

3

$f(x) < 0 \quad \forall x \leq 0$

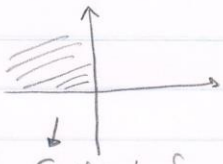

Gráfico de f

opción (d)

contradice la afirmación dada

4

$a > 0$
$b > 0$

$$(a+b) \cdot (a^{-1} + b^{-1}) = (a+b)\left(\frac{1}{a} + \frac{1}{b}\right) =$$

$$= (a+b)\left(\frac{b+a}{a \cdot b}\right) = \frac{(a+b)^2}{a \cdot b} = \frac{a^2 + 2ab + b^2}{a \cdot b} =$$

$$= \frac{a^2}{a \cdot b} + \frac{2ab}{a \cdot b} + \frac{b^2}{a \cdot b} = \frac{a}{b} + 2 + \frac{b}{a}$$

$$= 2 + \underbrace{\left(\frac{a}{b} + \frac{b}{a}\right)}_{> 0}$$ $\Rightarrow$ Tengo que probar si es mayor o menor que 2

Supongo que: $\frac{a}{b} + \frac{b}{a} < 2$

$$\frac{a}{b} + \frac{b}{a} < 2 \iff \frac{a^2+b^2}{\underbrace{a \cdot b}_{>0}} < 2 \iff a^2+b^2 < 2a \cdot b$$

$$\iff a^2+b^2-2ab < 0 \iff \underbrace{(a-b)^2}_{\geq 0} < 0 \quad \text{Contradicción}$$

$\Rightarrow$ Fue falso suponer que $\frac{a}{b} + \frac{b}{a} < 2$ $\therefore$

$\frac{a}{b} + \frac{b}{a} \geq 2$ Esto nos lleva a que:

$$(a+b)(a^{-1}+b^{-1}) = \ldots = 2 + \underbrace{\left(\frac{a}{b} + \frac{b}{a}\right)}_{\geq 2} \geq 4$$

opción (d)

5 $$\frac{1}{x(x-1)} - \frac{1}{(x-1)^2} = \frac{x-1-x}{x(x-1)^2} = \frac{-1}{x(x-1)^2}$$

opción (b)

6 $$\left(\frac{1}{6}\right)^{1/3} \cdot \left(\frac{32}{3}\right)^{-1/3} = \left(\frac{1}{6}\right)^{1/3} \cdot \left(\frac{3}{32}\right)^{1/3} = \sqrt[3]{\frac{1}{\cancel{6}_{2}} \cdot \frac{\cancel{3}^{1}}{32}} = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$

opción (b)

7 $4 \leq a \leq 40$

$5 \leq b \leq 12 \Rightarrow \frac{1}{5} \geq \frac{1}{b} \geq \frac{1}{12}$

$a \leq 40 \Rightarrow \frac{a}{5} \leq \frac{40}{5} \Rightarrow \frac{a}{5} \leq 8$ $*_1$

$\frac{1}{12} \leq \frac{1}{b} \leq \frac{1}{5} \xrightarrow{a>0} \frac{a}{12} \leq \frac{a}{b} \leq \frac{a}{5}$ $*_2$

Recuerden: Si $a<0$, la desigualdad cambia de sentido

$4 \leq a \Rightarrow \frac{4}{12} \leq \frac{a}{12} \Rightarrow \frac{1}{3} \leq \frac{a}{12}$ $*_3$

$$\underbrace{\frac{1}{3} \leq \frac{a}{12} \leq \frac{a}{b} \leq \frac{a}{5}}_{*_3 \quad *_2 \quad *_1} \leq 8 \quad \therefore \frac{1}{3} \leq \frac{a}{b} \leq 8$$

opción (b)

Otro camino — Teorema

H
- $a \leq b \leq c$
- $d \leq e \leq f$
- $a, b, c, d, e, f \in \mathbb{R}^+$

T $\frac{a}{f} \leq \frac{b}{e} \leq \frac{c}{d}$

Dem. $a \leq b \leq c$

$d \leq e \leq f \Rightarrow \frac{1}{d} \geq \frac{1}{e} \geq \frac{1}{f}$.

- $b \leq c \underset{d>0}{\Rightarrow} \frac{b}{d} \leq \frac{c}{d}$ $*_1$

- $\frac{1}{d} \geq \frac{1}{e} \geq \frac{1}{f} \underset{b>0}{\Longrightarrow} \frac{b}{d} \geq \frac{b}{e} \geq \frac{b}{f}$ $*_2$

- $a \leq b \underset{f>0}{\Longrightarrow} \frac{a}{f} \leq \frac{b}{f}$. $*_3$

$\therefore \underbrace{\frac{a}{f} \leq \frac{b}{f}}_{*_3} \leq \underbrace{\frac{b}{e} \leq \frac{b}{d}}_{*_2} \leq \underbrace{\frac{c}{d}}_{*_1} \Rightarrow \boxed{\frac{a}{f} \leq \frac{b}{e} \leq \frac{c}{d}}$

Luego de saber esta propiedad, podemos hacer el ej 7 así:

$4 \leq a \leq 40$
$5 \leq b \leq 12$ $\xrightarrow[\text{anterior}]{\text{por teorema}}$ $\frac{4}{12} \leq \frac{a}{b} \leq \frac{40}{5}$

$\Rightarrow \frac{1}{3} \leq \frac{a}{b} \leq 8$

Opción (b)