

Fórmula fundamental: $(\operatorname{sen} x)^2 + (\operatorname{cos} x)^2 = 1$

$$\operatorname{sen} 2x = 2 \cdot \operatorname{sen} x \cdot \operatorname{cos} x$$

$$\operatorname{cos} 2x = (\operatorname{cos} x)^2 - (\operatorname{sen} x)^2$$

11.2. TABLA DE FÓRMULAS TRIGONOMÉTRICAS

$$\begin{aligned} \operatorname{sen} \alpha &= \operatorname{cos} (90 - \alpha) \\ \operatorname{cos} \alpha &= \operatorname{sen} (90 - \alpha) \\ \operatorname{tg} \alpha &= \operatorname{cotg} (90 - \alpha) \end{aligned}$$

$$\begin{aligned} \operatorname{sen} \alpha &= -\operatorname{cos} (90 + \alpha) \\ \operatorname{cos} \alpha &= \operatorname{sen} (90 + \alpha) \\ \operatorname{tg} \alpha &= -\operatorname{cotg} (90 + \alpha) \end{aligned}$$

$$\begin{aligned} \operatorname{sen} \alpha &= -\operatorname{sen} (-\alpha) \\ \operatorname{cos} \alpha &= \operatorname{cos} (-\alpha) \\ \operatorname{tg} \alpha &= -\operatorname{tg} (-\alpha) \end{aligned}$$

$$\begin{aligned} \operatorname{sen} \alpha &= \operatorname{sen} (180 - \alpha) \\ \operatorname{cos} \alpha &= -\operatorname{cos} (180 - \alpha) \\ \operatorname{tg} \alpha &= -\operatorname{tg} (180 - \alpha) \end{aligned}$$

$$\begin{aligned} \operatorname{sen} \alpha &= \operatorname{sen} (2k\pi + \alpha) \quad k \in \mathbb{Z} \\ \operatorname{cos} \alpha &= \operatorname{cos} (2k\pi + \alpha) \quad k \in \mathbb{Z} \\ \operatorname{tg} \alpha &= \operatorname{tg} (2k\pi + \alpha) \quad k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \operatorname{sen} \alpha &= -\operatorname{sen} (180 + \alpha) \\ \operatorname{cos} \alpha &= -\operatorname{cos} (180 + \alpha) \\ \operatorname{tg} \alpha &= \operatorname{tg} (180 + \alpha) \end{aligned}$$

SUMA Y DIFERENCIA DE ÁNGULOS

$$\begin{aligned} \operatorname{sen} (\alpha + \beta) &= \operatorname{sen} \alpha \cdot \operatorname{cos} \beta + \operatorname{sen} \beta \cdot \operatorname{cos} \alpha \\ \operatorname{sen} (\alpha - \beta) &= \operatorname{sen} \alpha \cdot \operatorname{cos} \beta - \operatorname{sen} \beta \cdot \operatorname{cos} \alpha \\ \operatorname{cos} (\alpha + \beta) &= \operatorname{cos} \alpha \cdot \operatorname{cos} \beta - \operatorname{sen} \beta \cdot \operatorname{sen} \alpha \\ \operatorname{cos} (\alpha - \beta) &= \operatorname{cos} \alpha \cdot \operatorname{cos} \beta + \operatorname{sen} \beta \cdot \operatorname{sen} \alpha \\ \operatorname{tg} (\alpha + \beta) &= \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta} \\ \operatorname{tg} (\alpha - \beta) &= \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg} \beta} \end{aligned}$$

ÁNGULOS TRIPLES

$$\begin{aligned} \operatorname{sen} 3\alpha &= 3 \cdot \operatorname{sen} \alpha - 4(\operatorname{sen} \alpha)^3 \\ \operatorname{cos} 3\alpha &= 4(\operatorname{cos} \alpha)^3 - 3 \cdot \operatorname{cos} \alpha \\ \operatorname{tg} 3\alpha &= \frac{3 \operatorname{tg} \alpha - (\operatorname{tg} \alpha)^3}{1 - 3(\operatorname{tg} \alpha)^2} \end{aligned}$$

ÁNGULOS DOBLES

$$\begin{aligned} \operatorname{sen} 2\alpha &= 2 \cdot \operatorname{sen} \alpha \cdot \operatorname{cos} \alpha \\ \operatorname{cos} 2\alpha &= (\operatorname{cos} \alpha)^2 - (\operatorname{sen} \alpha)^2 \\ \operatorname{tg} 2\alpha &= \frac{2 \cdot \operatorname{tg} \alpha}{1 - (\operatorname{tg} \alpha)^2} \end{aligned}$$

FÓRMULAS DE FACTOREO

$$\begin{aligned} \operatorname{sen} \alpha + \operatorname{sen} \beta &= 2 \operatorname{sen} \left(\frac{\alpha + \beta}{2} \right) \operatorname{cos} \left(\frac{\alpha - \beta}{2} \right) \\ \operatorname{sen} \alpha - \operatorname{sen} \beta &= 2 \operatorname{cos} \left(\frac{\alpha + \beta}{2} \right) \operatorname{sen} \left(\frac{\alpha - \beta}{2} \right) \\ \operatorname{cos} \alpha + \operatorname{cos} \beta &= 2 \operatorname{cos} \left(\frac{\alpha + \beta}{2} \right) \operatorname{cos} \left(\frac{\alpha - \beta}{2} \right) \\ \operatorname{cos} \alpha - \operatorname{cos} \beta &= -2 \operatorname{sen} \left(\frac{\alpha + \beta}{2} \right) \operatorname{sen} \left(\frac{\alpha - \beta}{2} \right) \\ \operatorname{tg} \alpha + \operatorname{tg} \beta &= \frac{\operatorname{sen} (\alpha + \beta)}{\operatorname{cos} \alpha \cdot \operatorname{cos} \beta} \\ \operatorname{tg} \alpha - \operatorname{tg} \beta &= \frac{\operatorname{sen} (\alpha - \beta)}{\operatorname{cos} \alpha \cdot \operatorname{cos} \beta} \end{aligned}$$

ÁNGULOS MITAD

En estas fórmulas, delante del signo de raíz cuadrada deben ponerse los signos + o - según en qué cuadrante se encuentre el ángulo.

$$\operatorname{sen} \left(\frac{\alpha}{2} \right) = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\operatorname{cos} \left(\frac{\alpha}{2} \right) = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\operatorname{tg} \left(\frac{\alpha}{2} \right) = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{1 - \cos \alpha}{\operatorname{sen} \alpha}$$

ÁNGULOS CUÁDRUPLES

$$\operatorname{sen} 4\alpha = 8(\cos \alpha)^3 \cdot \operatorname{sen} \alpha - 4 \cdot \cos \alpha \cdot \operatorname{sen} \alpha$$

$$\operatorname{cos} 4\alpha = 8(\cos \alpha)^4 - 8(\cos \alpha)^2 + 1$$

$$\operatorname{tg} 4\alpha = \frac{4 \cdot \operatorname{tg} \alpha - 4 \cdot (\operatorname{tg} \alpha)^3}{1 - 6 \cdot (\operatorname{tg} \alpha)^2 + (\operatorname{tg} \alpha)^4}$$

TANGENTE DEL ARCO MITAD

$$\operatorname{sen} \alpha = \frac{2 \cdot \operatorname{tg} \left(\frac{\alpha}{2} \right)}{1 + \left(\operatorname{tg} \left(\frac{\alpha}{2} \right) \right)^2}$$

$$\operatorname{cos} \alpha = \frac{1 - \left(\operatorname{tg} \left(\frac{\alpha}{2} \right) \right)^2}{1 + \left(\operatorname{tg} \left(\frac{\alpha}{2} \right) \right)^2}$$

PRODUCTO DE FUNCIONES

$$\operatorname{sen} \alpha \cdot \operatorname{sen} \beta = \frac{\operatorname{cos} (\alpha - \beta) - \operatorname{cos} (\alpha + \beta)}{2}$$

$$\operatorname{cos} \alpha \cdot \operatorname{cos} \beta = \frac{\operatorname{cos} (\alpha - \beta) + \operatorname{cos} (\alpha + \beta)}{2}$$

$$\operatorname{sen} \alpha \cdot \operatorname{cos} \beta = \frac{\operatorname{sen} (\alpha - \beta) + \operatorname{sen} (\alpha + \beta)}{2}$$

**VALORES
EXACTOS
DE LAS
FUNCIONES
SENO
COSENO
TANGENTE**

Angulo α en grados	Angulo α en radianes	sen α	cos α	tg α
0°	0	0	1	0
15°	$\frac{\pi}{12}$	$\frac{(\sqrt{6}-\sqrt{2})}{4}$	$\frac{(\sqrt{6}+\sqrt{2})}{4}$	$2-\sqrt{3}$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
75°	$\frac{5\pi}{12}$	$\frac{(\sqrt{6}+\sqrt{2})}{4}$	$\frac{(\sqrt{6}-\sqrt{2})}{4}$	$2+\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	\neq
105°	$\frac{7\pi}{12}$	$\frac{(\sqrt{6}+\sqrt{2})}{4}$	$\frac{-\sqrt{6}-\sqrt{2}}{4}$	$-(2+\sqrt{3})$
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
165°	$\frac{11\pi}{12}$	$\frac{(\sqrt{6}-\sqrt{2})}{4}$	$\frac{-\sqrt{6}-\sqrt{2}}{4}$	$-(2-\sqrt{3})$
180°	π	0	-1	0
195°	$\frac{13\pi}{12}$	$\frac{-\sqrt{6}-\sqrt{2}}{4}$	$\frac{-\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
255°	$\frac{17\pi}{12}$	$\frac{-\sqrt{6}+\sqrt{2}}{4}$	$\frac{-\sqrt{6}-\sqrt{2}}{4}$	$(2+\sqrt{3})$
270°	$\frac{3\pi}{2}$	-1	0	\neq
285°	$\frac{19\pi}{12}$	$\frac{-\sqrt{6}+\sqrt{2}}{4}$	$\frac{(\sqrt{6}-\sqrt{2})}{4}$	$-(2+\sqrt{3})$
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
345°	$\frac{23\pi}{12}$	$\frac{-\sqrt{6}-\sqrt{2}}{4}$	$\frac{(\sqrt{6}+\sqrt{2})}{4}$	$-(2-\sqrt{3})$
360°	2π	0	1	0